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Stability Analysis of Quadruped-imitating Walking Robot Based on Inverted Pendulum Model

Yongming Wang, Xuetan Xu, Song Song
(School of Mechanical Engineering, Anhui University of Technology, Ma'anshan, Anhui Province, China)

ABSTRACT
A new kind of quadruped-imitating walking robot is designed, which is composed of a body bracket, leg brackets and walking legs. The walking leg of the robot is comprised of a first swiveling arm, a second swiveling arm and two striding leg rods. Each rod of the walking leg is connected by a rotary joint, and is directly controlled by the steering gear. The walking motion is realized by two striding leg rods alternately contacting the ground. Three assumptions are put forward according to the kinematic characteristics of the quadruped-imitating walking robot, and then the centroid equation of the robot is established. On this basis, this paper simplifies the striding process of the quadruped-imitating walking robot into an inverted pendulum model with a constant fulcrum and variable pendulum length. According to the inverted pendulum model, the stability of the robot is not only related to its centroid position, but also related to its centroid velocity. Takes two typical movement cases for example, such as walking on flat ground and climbing the vertical obstacle, the centroid position, velocity curves of the inverted pendulum model are obtained by MATLAB simulations. The results show that the quadruped-imitating walking robot is stable when walking on flat ground. In the process of climbing the vertical obstacle, the robot also can maintain certain stability through real-time control adjusted by the steering gears.
***Keywords* –**walking robot, stability, inverted pendulum model, motion equation, simulation analysis

I. INTRODUCTION

With the development of science and technology, the application of the walking robot is more and more widely. The motion stability of the walking robot is a hot issue concerned by scholars both at home and abroad. For example, based on the principle of stable cone, the stability of a wheel legged robot was evaluated by Tian Hai-bo [1]. Aiming at the point contact biped robot, its stability of the upright posture was studied by Sheng Tao [2]. Hybrid dynamic model of the three-link planar biped robot was established by Song Xian-xi [3], and the motion stability of the robot was analyzed by using Poincare mapping method. Based on a two-point-foot walking pattern, an inverted pendulum mode of the biped walking robot was established, and the stability of the walking process was analyzed by using Poincare mapping method [4]. In order to make the robot walk stably, Hu Jin-dong [5] used the centroid Jacobi matrix to achieve the centroid compensation. The static stability of the eight-legged robot was described by using the normalized energy stability margin method, and a mathematical model of static stability margin in complex environment was established Wang Li-quan [6]. The stability criterion of zero moment point (ZMP) was put forward by Vukobratovic[7], which is the basic method of walking stability analysis of biped robot. M.Wisse, a Holland scholar come from Technische Universiteit Delft, researched the stability of the robot from the viewpoint of dynamic system [8]. Kang [9] utilized the effective mass center (EMC) method to verify the stability of walking robot. From the perspective of energy consumption, Roy [10] studied the effect of the steering gait on the motion stability of the hexapod robot.

This paper studies a kind of quadruped-imitating walking robot, which walking motion is realized by two striding leg rods alternately contacting the ground. Three assumptions are put forward according to the kinematic characteristics of the quadruped-imitating walking robot. On this basis, the striding process of the robot is simplified into an inverted pendulum model with a constant fulcrum and variable pendulum length, and the factors affecting its stability are analyzed.

II. THE PRINCIPLE OF THE QUADRUPED-IMITATING WALKING ROBOT

The quadruped-imitating walking robot is composed of a body bracket, leg brackets and walking legs, and its principle is shown in Figure 1. Among them, the walking leg is comprised of a first swiveling arm, a second swiveling arm and two striding leg rods, wherein, all the leg rods are connected by rotary joints, and the rotary joints are driven by steering gears directly. The first swiveling arm, the second swiveling arm and the striding leg rod rotate at the speed ratio of 4:2:1, the two striding leg rods contact the ground alternately to complete the walking motion. The two striding leg rods at the same side are perpendicular. In order to control the steering of the quadruped-imitating walking robot, the leg bracket and the body bracket is connected by a rotary joint, which is driven by the steering gear.

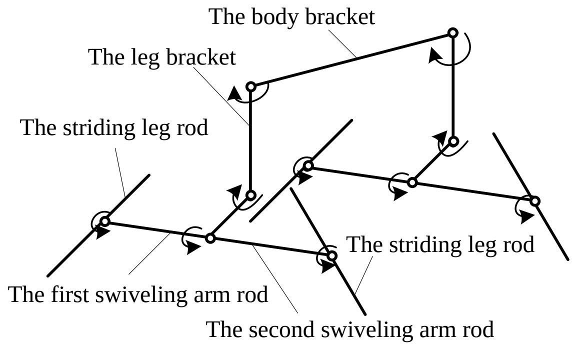

Fig.1 The principle of the walking robot

III. THE CENTROID EQUATION OF THE QUADRUPED-IMITATING WALKING ROBOT

The walking motion of the quadruped-imitating walking robot is realized by the fore and rear striding leg rods contact the ground alternately, so the centroid position of the robot is changing. Considering that the walking process of the quadruped-imitating walking robot is periodic, so takes one striding process for stability analysis.

A complete walking period includes two phases: four-leg supporting phase and two-leg supporting phase. The stability of the four-leg support phase is well, but the stability of the two-leg support phase is poor, so only the two-leg support phase is studied.

According to its kinematic characteristics, three assumptions are put forward.

1) The mass of each rod is concentrated in its centroid;

2) The contact process between the striding leg rod and the ground is realized by a rotation around a fulcrum, and the slippage is ignored;

3) Only consider about robots' movement in the vertical and fore-and-aft direction, and ignore the movement in the left-right direction.

As shown in Figure 2, according to the motion principle of the quadruped-imitating walking robot, in accordance with the right-hand rule to establish its coordinate system, the touchdown point of right striding leg rod is defined as the coordinate origin *O*, the forward direction is *X*-direction and the vertical direction is *Z*-direction. O_i represents the centroid of the rod *i*, *i* = 1, 2, …19, wherein, 1 and 11 are the first swiveling arms; 2 and 12 are the second swiveling arms; 3, 4, 13 and 14 are the striding leg rods; 5, 6, 7, 8, 15, 16, 17 and 18 are feet at the end of the striding leg rods, which are rubber and their mass are ignored; 9 and 19 are leg brackets; 10 is the body bracket. (x_i, y_i, z_i) represent the coordinates of the centroid of rod *i*.

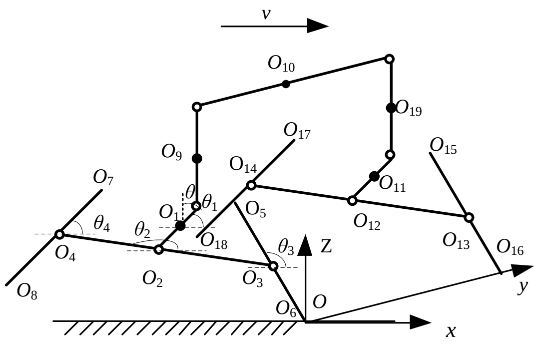

Fig.2 The coordinate system of the walking robot

The initial position of the robot is that the first swiveling arm is in vertical pose. θ_1, θ_2, θ_3 and θ_4 indicate the angles between the positive *X*-direction and the first swiveling arm, the second swiveling arm, the fore striding leg rod and the rear striding leg rod respectively. According to the coordinate system established in Figure 2, when the first swiveling arm has rotated θ angle from the initial position, the following equations are easy to get.

$$\begin{cases} \theta_1 = \dfrac{\pi}{2} - \theta \\ \theta_2 = \pi - \dfrac{\theta}{2} \\ \theta_3 = \dfrac{3\pi}{4} - \dfrac{\theta}{2} \\ \theta_4 = \dfrac{\pi}{4} - \dfrac{\theta}{4} \end{cases} \tag{1}$$

It is assumed that l_1, $2l_2$, $2l_3$, l_4 and l_5 are indicate the length of the first swiveling arm, the second swiveling arm, the striding leg rod, the leg bracket and the body bracket respectively. According to the coordinate system established in Figure 2, the centroid equations of rods are as follows (without considering Y-coordinates):

$$\begin{cases} x_1 = \dfrac{1}{2} l_1 \cos\theta_1 + l_2 \cos\theta_2 + l_3 \cos\theta_3 \\ z_1 = \dfrac{1}{2} l_1 \sin\theta_1 + l_2 \sin\theta_2 + l_3 \sin\theta_3 \end{cases} \tag{2}$$

$$\begin{cases} x_2 = l_2 \cos\theta_2 + l_3 \cos\theta_3 \\ z_2 = l_2 \sin\theta_2 + l_3 \sin\theta_3 \end{cases} \tag{3}$$

$$\begin{cases} x_3 = l_3 \cos\theta_3 \\ z_3 = l_3 \sin\theta_3 \end{cases} \tag{4}$$

$$\begin{cases} x_4 = 2l_2 \cos\theta_2 + l_3 \cos\theta_3 \\ z_4 = 2l_2 \sin\theta_2 + l_3 \sin\theta_3 \end{cases} \tag{5}$$

$$\begin{cases} x_5 = 2l_3 \cos\theta_3 \\ z_5 = 2l_3 \sin\theta_3 \end{cases} \tag{6}$$

$$\begin{cases} x_6 = 0 \\ z_6 = 0 \end{cases} \tag{7}$$

$$\begin{cases} x_7 = l_3 \cos\theta_4 + 2l_2 \cos\theta_2 + l_3 \cos\theta_3 \\ z_7 = l_3 \sin\theta_4 + 2l_2 \sin\theta_2 + l_3 \sin\theta_3 \end{cases} \tag{8}$$

$$\begin{cases} x_8 = 2l_2 \cos\theta_2 + l_3 \cos\theta_3 - l_3 \cos\theta_4 \\ z_8 = 2l_2 \sin\theta_2 + l_3 \sin\theta_3 - l_3 \sin\theta_4 \end{cases} \tag{9}$$

$$\begin{cases} x_9 = l_1 \cos\theta_1 + l_2 \cos\theta_2 + l_3 \cos\theta_3 \\ z_9 = l_1 \sin\theta_1 + l_2 \sin\theta_2 + l_3 \sin\theta_3 + l_4/2 \end{cases} \tag{10}$$

$$\begin{cases} x_{10} = l_1 \cos\theta_1 + l_2 \cos\theta_2 + l_3 \cos\theta_3 \\ z_{10} = l_1 \sin\theta_1 + l_2 \sin\theta_2 + l_3 \sin\theta_3 \end{cases} \tag{11}$$

$$x_i = x_j, z_i = z_j, i = 11,\ 12\ldots19, j = 1,\ 2,\ \ldots\ 9 \tag{12}$$

According to the coordinates of the centroid of each rod given in equation (1) to (12), the centroid coordinates' expressions of the quadruped-imitating walking robot can be obtained.

$$\begin{cases} x = \dfrac{\sum\limits_{i=1}^{19} m_i x_i}{\sum\limits_{i=1}^{19} m_i} \\ z = \dfrac{\sum\limits_{i=1}^{19} m_i z_i}{\sum\limits_{i=1}^{19} m_i} \end{cases} \tag{13}$$

IV. THE INVERTED PENDULUM MODEL OF THE QUADRUPED-IMITATING WALKING ROBOT

Based on the above assumptions, the quadruped-imitating walking robot can be simplified as an inverted pendulum model with a constant fulcrum and variable pendulum length, as shown in Figure 3. There are two cases of centroid pose of the inverted pendulum model. In Figure 3 (a), there is a minor deviation between the centroid position and the supporting point *O*. If the centroid position of the robot moves toward the supporting point *O*, then the robot can recover balance by adjusting the centroid position through the striding movement before the robot falls down, so its stability is high. If the centroid position of the robot moves away from the supporting point *O*, and the deviation is increasing, then the robot may not be able to adjust in time and then will fall down. In Figure 5 (b), the centroid position deviates from the supporting point *O* too far, and which is beyond the adjustment range of the robot, therefore, it will be fall down.

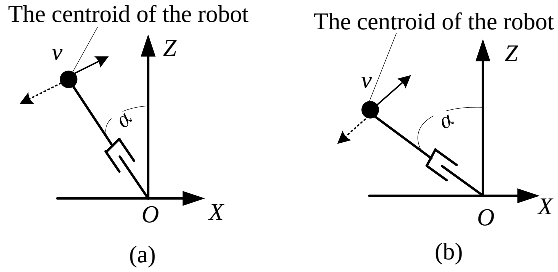

Fig.3 The inverted pendulum model of quadruped-imitating walking robot

As shown in Figure 4, if the centroid of the quadruped-imitating walking robot is located at the upper right of the supporting point *O*, the robot has a tendency to fall down in the positive direction of *X*-axis. At this time, the robot's two rear striding leg rods have rotated to the front of the supporting point *O*, and prepare to contact the ground at the touchdown point *A*. In this case, to maintain the stability of the robot, it is necessary to ensure that the centroid of the robot is located at the left of the touchdown point *A*.

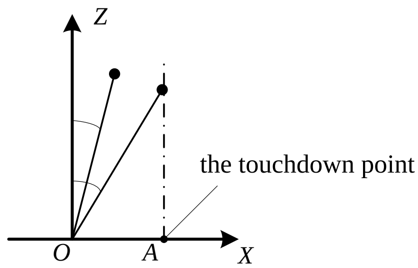

Fig.4 The position relationship between the centroid of the robot and the touchdown point

Through the analysis of the inverted pendulum model of the robot, the factors affecting the movement stability are not only related to the centroid position of the robot, but also related to the centroid velocity of the robot. If the centroid of the robot is closer to the supporting point *O* and its movement direction is opposite to the direction of tumble tendency, then the stability of the robot is better.

V. STABILITY ANALYSIS OF THE ROBOT WHEN WALKING ON FLAT GROUND

According to the inverted pendulum model of the robot mentioned above, the position and velocity of the centroid can be simulated by using MATLAB software, and the simulation parameters of the robot are shown in table 1.

Table.1 Simulation parameters

Parameter name	Symbol	Value
The length of the first swiveling arm	l_1	0.05 m
The half-length of the second swiveling rod	l_2	0.08 m
The half-length of the striding leg rod	l_3	0.07 m
The length of the leg bracket	l_4	0.225 m
The length of the body bracket	l_5	0.45 m
The angular velocity of the first swiveling arm	ω	12 rpm

According to the equation (13) and Table 1, the centroid displacement curve of the quadruped-imitating walking robot is obtained by MATLAB simulation, as shown in Figure 5. In a stride process, the robot's centroid displacement in *X*-direction changes from -0.130m to 0.130m, the centroid displacement firstly increases from 0.135 m to 0.190 m in *Z*-direction, after moves across the coordinate origin point, the centroid displacement drops to 0.135m again.

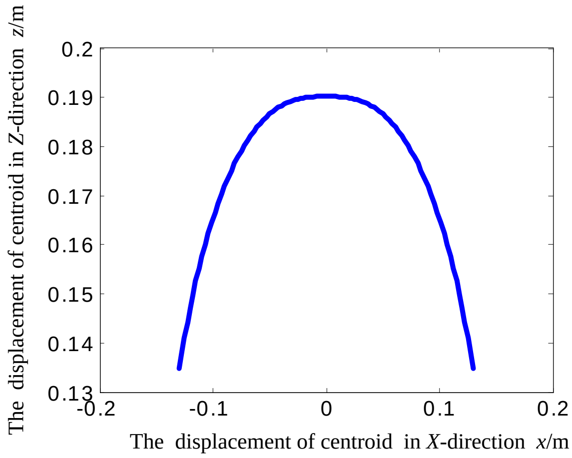

Fig.5 The centroid displacement curve of the robot when walking on flat ground

When the centroid of robot is on the negative half of *X-axis*, the robot has a tendency to fall to negative direction *X*-direction. The angle between the positive direction of *X-axis* and the beeline defined by the initial centroid position and the touchdown point *O* is largest, that is:

$$\alpha_{\max} = \arctan \frac{|x_0|}{z_0} \tag{14}$$

According to the simulation results of Figure 5, it can be known that x_0=-0.130m, z_0=0.135m, then according to equation (14), the angle $\alpha_{\max}$ is 43.9°. As the robot walks, the angle becomes smaller, and the distance between the centroid and the origin *O* decreases gradually, so the stability is pretty good.

The stability of the quadruped-imitating walking robot is also related to the velocity of the centroid. The velocity curve of the centroid in the *X*-direction can be obtained by MATLAB simulation, as shown in Figure 6. When the centroid of the robot is on the negative half *X-axis*, the velocity of the centroid in *X*-direction increases to the maximum of 0.0465m/s rapidly, and then decreases gradually. When the centroid of the robot is moving to the above of the origin *O*, its velocity decreases to the minimum of 0.0345 m/s. In this process, the velocity of the centroid is positive, that is, the robot is moving towards the opposite direction of the robot trends to fall down, so the stability of the robot is good in this process.

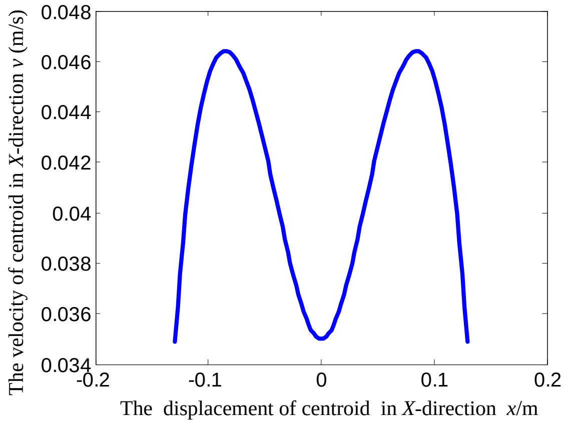

Fig.6 The velocity curve of the centroid in the *X*-direction

When the centroid of the robot is on the positive half X-axis, the velocity of the centroid in X-direction increases to the maximum of 0.0465m/s gradually, and then decreases rapidly, and the velocity of centroid is positive all the time. In this process, the two rear striding leg rods are moving on the positive half *X*-axis, that is, they are moving from origin *O* toward touchdown point A. According to the coordinate system shown in Figure 2, the displacement curve of the end point of the rear striding leg rod can be obtained as shown in Figure 7. The displacement of the end point of the rear striding leg rod in *X*-direction changes from -0.16m to 0.259m, and in Figure 5, the robot's centroid position changes from 0 to 0.130m. That is, the centroid is on the left of the touchdown point A, so it is stable when the robot walking on the flat ground.

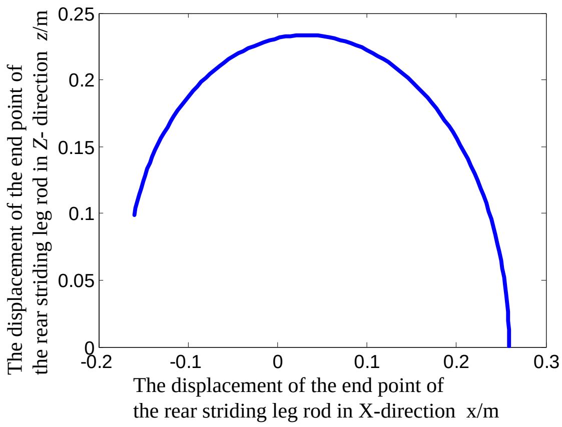

Fig.7 The displacement curve of the end point of the rear striding leg rod

VI. STABILITY ANALYSIS OF THE ROBOT WHEN CLIMBING THE VERTICAL OBSTACLE

When the robot climbs the vertical obstacle, its centroid equation can be established similar to that when the robot walks on flat ground. The centroid displacement curve of the quadruped-imitating walking robot when climbing the vertical obstacle is obtained through MATLAB simulation, as shown in Figure 8.

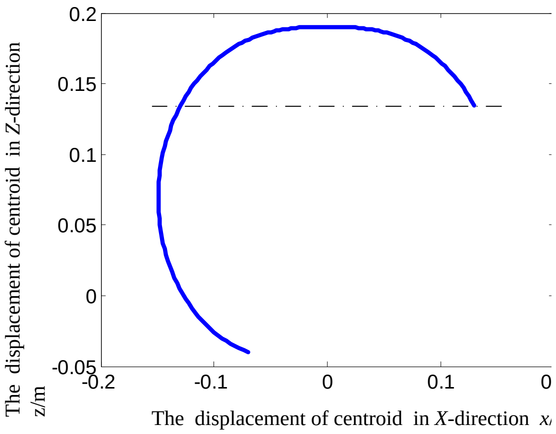

Fig.8 The centroid displacement curve of the robot when climbing the vertical obstacle

The centroid displacement of the robot in the *X*-direction ranges from -0.07m to 0.130m. The centroid displacement of the robot in Z-direction begins from -0.0398m, when the centroid of the robot is moving to the top of the origin *O*, it increases to the maximum of 0.190m in *Z*-direction, and after the centroid moves across the origin *O*, and the centroid displacement drops to 0.135m in the *Z*-direction. The curve above the dotted line in Figure 8 is the same as that shown in Figure 5, it illustrates that the robot has completed climbing the vertical obstacle.

Assumes that α_0 is the angle between the positive direction of *Z*-axis and the beeline defined by the initial centroid and the origin *O*, and its value is:

$$\alpha_0 = \pi\text{-}\arctan\left|\frac{x_0}{z_0}\right| \tag{15}$$

According to the simulation results of Figure 8, it can be known that x_0=-0.07m, z_0=-0.0398m, and α_0=119.6°. At the beginning of climbing the vertical obstacle, the centroid of the robot is on the negative half *X*-axis, the distance between the centroid and the original point *O* is large, so the robot has the tendency to fall down.

The stability of the quadruped-imitating walking robot is also related to the velocity of the centroid. When the robot climbs the vertical obstacle, the velocity curve of the centroid in the *X*-direction is obtained through MATLAB simulation, as shown in Figure 9.

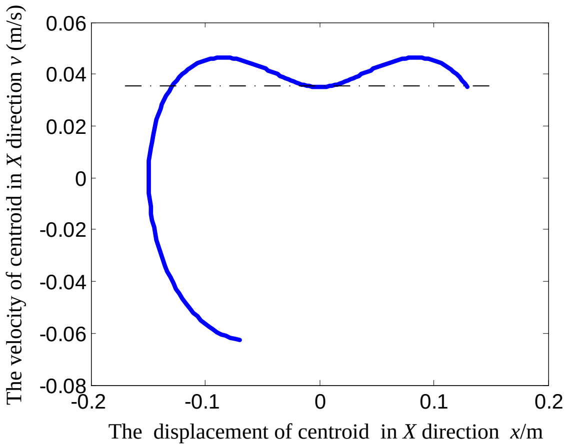

Fig.9 The velocity curve of the centroid in the *X*-direction

The minimum value of the velocity of the robot's centroid in the *X*-direction is -0.0625m/s, and the maximum one is 0.0464 m/s. The curve above the dotted line in Figure 9 is the same as that shown in Figure 6, it illustrates that the robot has completed climbing the vertical obstacle. In the process of the robot climbing the vertical obstacle, the displacement of the robot's centroid in the *X*-direction changes from -0.07m to 0.1494m, and the value of velocity is negative, the centroid of robot moves away from the touchdown point A, so its stability is poor. Due to all rods of the walking leg are rotated by the steering gears directly, when the robot is going to out of balance, the centroid of robot is moving toward the touchdown point A by changing the output torque of the steering gears. When the velocity of the centroid changes to a positive value, the stability of the robot is improved. When the velocity of the centroid increases in X-direction, it will move towards the touchdown point A and the robot will recover to the gait that walking on flat ground eventually.

VII. CONCLUSION

According to the kinematic characteristics of the quadruped-imitating walking robot, the striding process of the robot is simplified into an inverted pendulum model with a constant fulcrum and variable pendulum length, and the factors affecting its stability are analyzed.

1) The factors that affect the movement stability of the robot are not only related to the centroid position, but also related to the velocity of the centroid. If the centroid is closer to the supporting point and its movement direction is opposite to the direction of the tumble tendency of the robot, then the stability of the robot is better.

2) Through the analysis of the centroid position and velocity of the robot based on the inverted pendulum model, it is proved that the stability of the robot walking on flat ground is better than that climbing the vertical obstacle. In the process of climbing the vertical obstacle, the robot can also maintain its stability through real-time adjustment of the steering gears.

VIII. ACKNOWLEDGEMENTS

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